


**OVERVIEW**

# Recent developments in multivariate wind and solar power forecasting

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**Abstract**

The intermittency of renewable energy sources, such as wind and solar, means that they require reliable and accurate forecasts to integrate properly into energy systems. This review introduces and examines a selection of state-of-the-art methods that are applied for multivariate forecasting of wind and solar power production. Methods such as conditional parametric and combined forecasting already see wide use in practice, both commercially and scientifically. In the context of multivariate forecasting, it is vital to model the dependence between forecasts correctly. In recent years, reconciliation of forecasts to ensure coherency across spatial and temporal aggregation levels has shown great promise in increasing the accuracy of renewable energy forecasts. We introduce the methodology used for forecast reconciliation and review some recent applications for wind and solar power forecasting. Many forecasters are beginning to see the benefit of the greater information provided by probabilistic forecasts. We highlight stochastic differential equations as a method for probabilistic forecasting, which can also model the dependence structure. Lastly, we discuss forecast evaluation and how choosing a proper approach to evaluation is vital to avoid misrepresenting forecasts.

This article is categorized under:

Climate and Environment > Net Zero Planning and Decarbonization  
 Sustainable Energy > Solar Energy  
 Sustainable Energy > Wind Energy

**KEYWORDS**

forecast evaluation, forecast reconciliation, solar power forecasting, stochastic differential equations, wind power forecasting

**1 | INTRODUCTION**

Renewable energy sources are increasingly looked to as the main solution to the ongoing climate crisis. As a result, the penetration of renewable energy sources has increased dramatically in recent years and is expected to increase even

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further in the coming decades. This presents a new challenge as wind turbines and solar panels only produce power when the wind blows and the sun shines. The stochastic nature of the main renewable energy sources means that they require reliable and accurate forecasts to integrate properly.

Power generation from intermittent sources is challenging because it disrupts conventional methods for planning and operating energy systems. Renewable power production varies and fluctuates on multiple time scales, forcing grid operators to adjust their day-ahead, hour-ahead, minute-ahead, and real-time operating procedures. Consequently, in areas with a high penetration of renewables, such as wind power in the Western part of Denmark and the Northern part of Germany, reliable forecasts are needed on all relevant temporal scales in order to ensure safe and economic operation of the power system.

Taking the Scandinavian area as an example, accurate wind and solar power forecasts are needed with different horizons to ensure:

- (up to a few hours) efficient and safe use of regulating power (spinning reserve), management of the distribution and transmission system, efficient trading in intraday markets,
- (12–36 h) efficient trading on the physical power exchange, and
- (days) optimal operation of assets, such as large combined heat and power (CHP) plants.

Forecasts are also needed for several spatial aggregation levels, for example, for planning of network operation and use of storage solutions. Again considering the Nordic region with area pricing of electricity, forecasts are typically needed for the total price area, the supply area, as well as on regional scale and for individual wind farms. In areas with nodal pricing, such as North America, forecasts linked to the physical layout of the network are very important.

A number of reviews have been published that cover the field of renewable energy forecasting. In a recent review, Hong et al. (2020) provide an outlook for the field as a whole, showing some of the directions being pursued and highlighting some of the challenges within the field. Most reviews of wind and solar power forecasting provide broad overviews of the methods being used (see, e.g., Costa et al. (2008), Giebel and Kariniotakis (2017), or Hanifi et al. (2020) for wind power forecasting).

In this review, we narrow the focus to recent developments within multivariate wind and solar power forecasting. We focus on practical methods that are useful in real-time production environments rather than computationally demanding offline methods. Multivariate methods are to be interpreted as forecasting multiple, interconnected variables simultaneously. This can be forecasts for multiple areas, multiple time horizons, or both. Forecasting can be further subdivided into point forecasting, that is, generating a single numeric prediction containing the expectation for that point, and probabilistic forecasting, where the possible predicted outcomes are assigned a probability, and the distribution of the predictions is reported (Haupt et al., 2019).

We follow this division and discuss both point and probabilistic forecasting. In the context of multivariate forecasting, it is vital to model the covariance structure, or more generally the interdependence structure, correctly. Within both categories, we highlight a method used for this purpose, which has seen recent development. We briefly introduce the required background before examining some of the recent applications related to wind and solar power forecasting.

Within the realm of point forecasting, we choose to highlight the method of forecast reconciliation, which has been successful in numerous applications for wind and solar power forecasting. Although most research so far has focused on reconciliation of point forecasts, we also briefly introduce recent extensions to probabilistic forecasts. For probabilistic forecasting, we highlight stochastic differential equations (SDEs) as a tool for modeling dependence structures. Some interesting work has recently been done related to wind and solar power forecasting using SDEs.

Lastly, we consider the importance of proper forecast evaluation. We introduce and discuss scoring rules for evaluation of both univariate and multivariate forecasts. The multivariate aspect increases the complexity of evaluating their performance. Choosing the right way of evaluating forecasts is vital to ensure that the forecasts are not misrepresented. We discuss how analysis of actual decision problems can be used to derive the forecast characteristics important for a given application and choose the right evaluation metric.

The outline of this review is as follows. We begin with a review of some of the current state-of-the-art methods used for wind and solar power forecasting in Section 2. In Section 3, we introduce reconciliation of hierarchical forecasts and discuss some recent applications to wind and solar power forecasting. SDEs are discussed as a tool for generating probabilistic forecasts in Section 4. Section 5 reviews some of the necessary tools for forecast evaluation of both point and probabilistic forecasts and discusses practical aspects of evaluating wind and solar power forecasts. Finally, Section 6 summarizes the methods that have been introduced and provides an outlook for future work.

## 2 | METHODS FOR POINT FORECASTING

In this section, we briefly outline some of the currently used state-of-the-art methods for forecasting of wind and solar power production. For basic operation, a forecast consists of a single value for each time point in the future. Naturally, it should be as accurate as possible and its delivery should be reliable. To achieve these goals, the forecasting system should use state-of-the-art numerical weather predictions (NWP) together with measurements of actual production from the site or region considered. If available, such a system can further benefit from measurements and forecasts of availability and curtailment.

Machine- and deep-learning approaches have seen a resurgence in recent years (see, e.g., Wang et al., 2019). Yet, statistical and physical methods are widely used due to their lower computational demand and their capabilities for adaptation and extrapolation. In a time-scale classification of wind power forecasting techniques, Soman et al. (2010) concluded that statistical methods, such as machine learning and autoregressive time series models without exogenous input, are useful for very short-term forecasting. For longer horizons, however, it is crucial to take advantage of NWP forecasts of, for example, wind speed and direction (Nielsen et al., 2002a). For medium-term predictions, a combination of local data with meteorological forecasts is needed.

Meteorological forecasts are often biased. Furthermore, meteorological ensemble-based methods have to be calibrated in order to be aligned with the observed uncertainty (Nielsen et al., 2006a). In the following, we will consider dynamical models formulated as linear regression models, which may include meteorological forecasts as exogenous input. These models enable an integrated method for bias correction and, depending on the selected explanatory variables, are suitable for point forecasting on all scales.

Regardless of the method used for forecasting, adaptive methods are needed in order to capture time-varying phenomena, such as variation due to changes in surroundings or wear of installed equipment. We consider both parametric and conditional parametric models. The non-parametric part of the conditional parametric models is able to account for phenomena that are difficult to specify as a parametric model. A classical example in wind power forecasting is that both the layout of a wind farm and spatial variation in the roughness of the surrounding terrain make it difficult, or even impossible, to suggest a parametric dependency on wind direction. In any case incorporating information from different sources (e.g., different meteorological forecasts) has proven useful (Nielsen et al., 2007).

### 2.1 | Forecasting using linear time series models

Let us consider a simple linear time series model describing the relationship between data for wind power,  $Y_t$ , and explanatory variables, such as locally measured wind speed and meteorological forecasts of wind speed, air temperature, and so forth. For short-term forecasting of renewable energy production, the autoregressive dynamics are important, and hence autoregressive linear time series models with exogenous input (ARX models) are useful. An ARX model for wind power generation can be written as

$$Y_t = \mathbf{x}_t^T \boldsymbol{\theta} + \varepsilon_t, \quad (1)$$

where  $\mathbf{x}_t$  contains lagged values of  $Y_t$  and the explanatory variables. The vector  $\boldsymbol{\theta}$  contains the unknown model parameters.

Assuming that the data available at time  $t$  is

$$Y_1, Y_2, \dots, Y_{t-1}, Y_t,$$

$$\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{t-1}, \mathbf{x}_t,$$

then the least-squares estimate of the parameter vector based on  $t$  observations is

$$\hat{\boldsymbol{\theta}}_t = \arg \min_{\boldsymbol{\theta}} \sum_{s=1}^t (Y_s - \mathbf{x}_s^T \boldsymbol{\theta})^2. \quad (2)$$

This estimate can be optimized for  $k$ -step prediction by replacing  $\mathbf{x}_s$  with  $\mathbf{x}_{s-k}$  in (2). Using the estimated parameters, future values of the wind power production  $Y_{t+k}$  can be forecasted as

$$\widehat{Y}_{t+k} = \mathbf{x}_t^T \widehat{\boldsymbol{\theta}}_t. \quad (3)$$

For linear time series models, such as the ARX model, the scale parameter or the variance of the  $k$ -step forecast error is constant (see Madsen, 2008). An example of a simple model for wind power forecasting belonging to this model class is given in Section 2.3.1.

## 2.2 | Adaptive forecasting

### 2.2.1 | Recursive least squares with exponential forgetting

For adaptive forecasting, using recursive least squares with a forgetting factor  $\lambda$ , the parameters are estimated using

$$\widehat{\boldsymbol{\theta}}_t = \operatorname{argmin}_{\boldsymbol{\theta}} \sum_{s=1}^t \lambda^{t-s} (Y_s - \mathbf{x}_s^T \boldsymbol{\theta})^2. \quad (4)$$

This is a weighted least-squares method, where the weight on older observations decays exponentially. The principles for generating forecasts are the same as before.

In a recursive framework, the formula for updating the estimate of  $\boldsymbol{\theta}_t$  is conveniently written as

$$\mathbf{R}_t = \mathbf{x}_t \mathbf{x}_t^T + \lambda \mathbf{R}_{t-1}, \quad (5)$$

$$\widehat{\boldsymbol{\theta}}_t = \widehat{\boldsymbol{\theta}}_{t-1} + \mathbf{R}_t^{-1} \mathbf{x}_t (Y_t - \mathbf{x}_t^T \widehat{\boldsymbol{\theta}}_{t-1}). \quad (6)$$

In the formula above, it is straightforward to replace the constant forgetting  $\lambda$  with a time-varying forgetting  $\lambda(t)$ . It is also possible to introduce a feedback from the forecast errors to a time-varying forgetting factor. As an example, it might be useful to increase the forgetting factor if large errors occur (see Madsen, 2008, for details).

For starting the recursions,  $\boldsymbol{\theta}_t$  can be initialized with a vector of zeroes. The covariance matrix  $\mathbf{R}_t$  can be initialized as an identity matrix times a large constant, which reflects the large uncertainty for the initial value of the parameters.

### 2.2.2 | Model-based adaptive forecasting

Instead of using a single forgetting factor, it is sometimes more appropriate to formulate a model for the unknown time-varying parameters. Using a state-space model formulation, it is possible to describe that some parameters are constant while other parameters exhibit large variations over time.

A simple state-space model for the time-varying parameters is

$$\boldsymbol{\theta}_{t+1} = \boldsymbol{\Phi} \boldsymbol{\theta}_t + \mathbf{e}_{1,t}, \quad V(\mathbf{e}_{1,t}) = \boldsymbol{\Sigma}_1, \quad (7)$$

$$Y_t = \mathbf{x}_t^T \boldsymbol{\theta}_t + \mathbf{e}_{2,t}, \quad V(\mathbf{e}_{2,t}) = \boldsymbol{\Sigma}_2, \quad (8)$$

where  $\mathbf{e}_{1,t}$  and  $\mathbf{e}_{2,t}$  are mutually independent Gaussian white noise processes. Note that if  $\boldsymbol{\Phi}$  is equal to the identity matrix, then the changes in the model parameters are described as a random walk process. A constant parameter can be defined by setting the appropriate variance of the system noise process to zero.

The Kalman filter can be used for tracking the parameters and the recursions, without the so-called update step, can be used to generate  $k$ -step forecasts (see, e.g., Madsen, 2008). The principles can easily be generalized to nonlinear and time-varying models. Similarly, other filtering techniques can be used. In Section 4, we will introduce continuous-discrete time state-space models formulated as partially observed stochastic differential equations for multi-step probabilistic forecasting.

### 2.3 | Conditional parametric forecasting

In the adaptive setting, the parameters vary in time. Another option, which is highly relevant for wind power forecasting, is that the parameters are functions of some (other) explanatory variables.

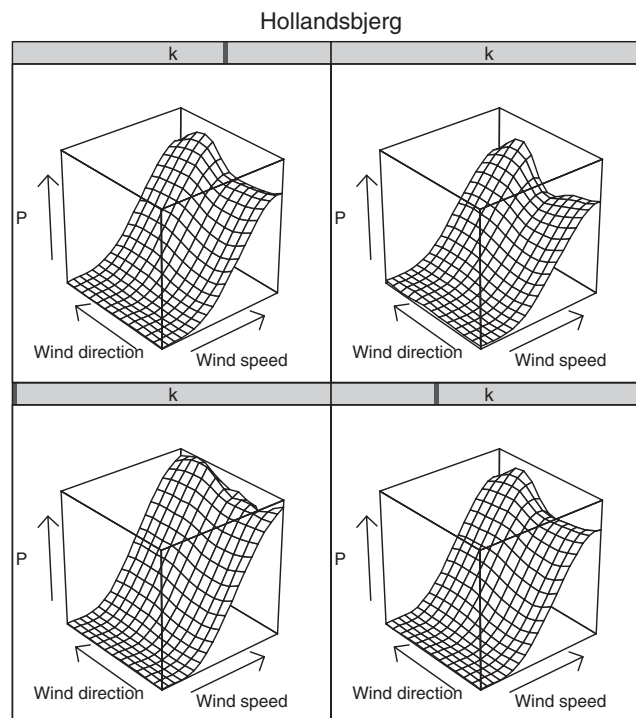
Considering the same setting as above, the conditional parametric model is written as

$$Y_t = \mathbf{x}_t^T \boldsymbol{\theta}(\mathbf{u}) + \varepsilon_t, \quad (9)$$

where the parameters depend on some additional explanatory variables  $\mathbf{u}$ . A classical example in wind power forecasting is a power curve for a wind farm, which includes a parametric dependency on wind speed and a nonparametric dependency on wind direction. The parametric dependency on wind speed is nonlinear in the parameters, which illustrates that the method can be readily extended to nonlinear functions. As an example, Figure 1 shows the estimated wind farm power curve for four different forecast horizons for the Høllandsbjerg wind farm in Denmark (Nielsen et al., 2002b).

The parameters are estimated locally around a fitting point  $\mathbf{u}$ . The locality is described by a Kernel function. Using this Kernel function, the local parameters of the conditional parametric model are given by

$$\hat{\boldsymbol{\theta}}(\mathbf{u}) = \arg \min_{\boldsymbol{\theta}} \sum_{s=1}^t W_s(\mathbf{u}) (Y_s - \mathbf{x}_s^T \boldsymbol{\theta})^2. \quad (10)$$



**FIGURE 1** Expected power production as a function of forecasted wind speed and direction for four different forecast horizons for the Høllandsbjerg wind farm in Denmark. From bottom left to top right, the power curves correspond to forecast horizons of 0 (analysis), 12, 24, and 36 h (Nielsen et al., 2002b)

The weights can be assigned as

$$W_s(\mathbf{u}) = w\left(\frac{\|\mathbf{u}_s - \mathbf{u}\|}{h(\mathbf{u})}\right), \quad (11)$$

where  $\|\cdot\|$  denotes the Euclidean norm,  $h(\mathbf{u})$  is the bandwidth used for the particular fitting point, and  $w(\cdot)$  is a weighting function taking non-negative arguments. The weighting may, for instance, be chosen as a tricube function

$$w(u) = \begin{cases} (1 - u^3)^3, & u \in [0; 1), \\ 0, & u \in [1; \infty), \end{cases} \quad (12)$$

to ensure that the weights are between zero and one.

Again the same formula for forecasting is used. It is straightforward to include adaptive estimation of the parameters in the conditional parametric model. For further details, see Nielsen et al. (2000) who used this for short-term wind power forecasting.

### 2.3.1 | Example: An ARX model for wind power forecasting

This example illustrates the application of a simple but useful model for  $k$ -step wind power forecasting, using all of the above-mentioned techniques. The model belongs to the class of linear ARX models introduced previously. The power curve predictions are used as input to an adaptively estimated dynamical model, which leads to the following  $k$ -steps ahead forecasts:

$$\hat{P}_{t+k|t} = a_1 p_t + a_2 p_{t-1} + b \hat{P}_{t+k|t}^{pc} + \sum_{i=1}^3 \left[ c_i^c \cos \frac{2i\pi h_{t+k}^{24}}{24} + c_i^s \sin \frac{2i\pi h_{t+k}^{24}}{24} \right] + m,$$

where  $p_t$  is observed power production,  $k \in [1; 48]$  (hours) is the forecast horizon,  $\hat{P}_{t+k|t}^{pc}$  is the power curve prediction, and  $h_{t+k}^{24}$  is the time of day. Finally,  $a_1, a_2, b, c_i^c, c_i^s, m$  ( $i = 1, 2, 3$ ) are time-varying parameters that are estimated adaptively. This class of models is discussed in Nielsen et al. (2002b), where the techniques are used for wind power forecasting for six different wind farms in Denmark and Spain.

## 2.4 | Combined forecasting

In some cases multiple forecasts of the same observation are available. For example, this can happen when the same model is fitted based on NWP's from different providers or when different models are used for forecasting the same data. In combined forecasting, different forecasts are combined in an optimal way based on the variance of each forecast error and the correlation between the error of the different forecasts. Combined forecasting is conceptually similar to Bayesian model averaging, where each model is weighted by its posterior evidence (see, e.g., Hoeting et al., 1999; Raftery et al., 2005).

Nielsen et al. (2007) considered wind power forecasts based on a number of different meteorological forecasts originating from three different global meteorological models. Although the wind power forecasts had fairly similar accuracy, they showed that combined forecasting led to approximately 10% improvement in accuracy compared to using only a single meteorological forecast provider.

Assuming  $J$  forecasts are given, Nielsen et al. (2007) proposed to combine them using

$$\hat{y}_c = \sum_{j=1}^J (\hat{y}_j - \mu_j) w_j. \quad (13)$$

The biases ( $\mu_j$ ) can be collected to get

$$\hat{y}_c = \mu_0 + \sum_{j=1}^J \hat{y}_j w_j, \quad (14)$$

where  $w_j$  is the weight of each forecast, which should be chosen in an optimal way. In order to interpret  $w_j$  as weights, the constraint  $\sum_{j=1}^J w_j = 1$  is imposed. Nielsen et al. (2007) derived the solution to the above problem by minimizing the variance of the combined forecast error based on the covariance matrix of the forecast errors of the original forecasts.

The above problem can be formulated as

$$\hat{y}_c = \mathbf{X}\boldsymbol{\beta}, \quad (15)$$

where  $\hat{y}_c$  is the collection of all combined forecasts (on the training set) and  $\mathbf{X}$  is a design matrix with the first column equal to one and the other columns equal to the individual forecasts. Linear constraints can be included directly by

$$\mathbf{B}\boldsymbol{\beta} = \mathbf{c}. \quad (16)$$

In the example above with  $\boldsymbol{\beta} = [\mu_0, w_1, \dots, w_J]$ , we would have  $\mathbf{B} = [0, 1, \dots, 1]$  and  $\mathbf{c} = 1$ .

#### 2.4.1 | Example: Combined forecasting of wind power production

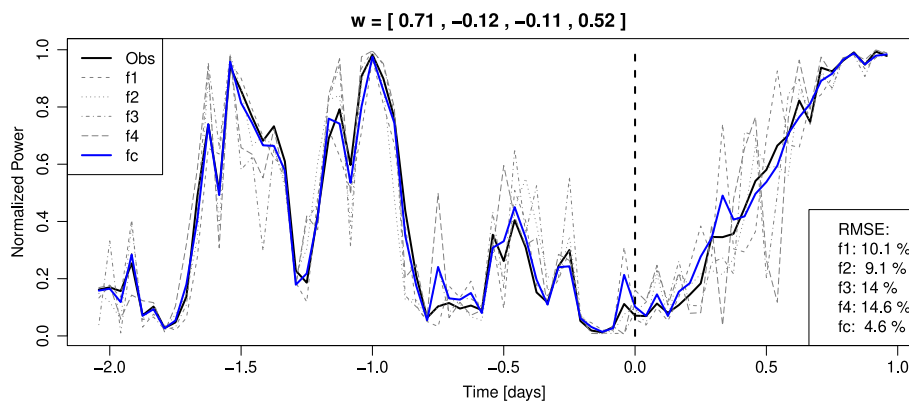
Figure 2 illustrates the principle of combined forecasting on a simulated data example, using the following system (unit of  $t$  being hours)

$$p_{norm,t} = \frac{e^{y_t}}{1 + e^{y_t}}, \quad (17)$$

$$y_t = \sin\left(\frac{2\pi t}{48}\right) - 1 + e_t, \quad (18)$$

$$e_t = 0.9e_{t-1} + v_t; \quad v_t \sim N(0, 1). \quad (19)$$

The individual forecasts are generated by



**FIGURE 2** Illustration of combined forecasting. In this simulated example, we have four individual forecasts with a covariance structure of the errors defined by (20)–(22) resulting in a combination of positive and negative weights. The training period is 50 days and the RMSE of the forecast is reduced by a factor of two compared to the best individual forecast (from 9.1% to 4.6%)

$$\hat{\mathbf{y}}_t \sim N(y_t \mathbf{1}, \boldsymbol{\Sigma}), \quad (20)$$

$$\hat{p}_{norm,t}^{ii} = \frac{e^{\hat{y}_t^i}}{1 + e^{\hat{y}_t^i}}, \quad (21)$$

with

$$\boldsymbol{\Sigma} = \frac{1}{16} \begin{bmatrix} 7 & 1 & -1 & -8 \\ 1 & 6 & 5 & 4 \\ -1 & 5 & 13 & 8 \\ -8 & 4 & 8 & 16 \end{bmatrix}. \quad (22)$$

Although the weights sum to one they are not guaranteed to be positive when the full covariance matrix is used in the construction of the weights. The improvement in the synthetic data example in terms of RMSE is a factor of around two compared to the best individual forecast.

Forecast combination has been used successfully for a number of applications, including wind power production, see, for example, Sanchez (2008) or Buhan and Cadirci (2015). The method is also a staple in most forecast competitions and tends to be used by the winning forecasters. As explained by Atiya (2020), as long as forecasts are diverse and of mostly equal performance, combining them is likely to result in an improved forecast.

### 3 | FORECAST RECONCILIATION

Many energy-related forecasting and decision problems involve multiple locations and/or time horizons that form a hierarchy. For example, power production has to meet expected demand over the next minutes, hours, days, and even longer horizons. When the sum of the short-term forecasts is not coherent with the longer-term forecasts this leads to suboptimal decision making.

When forecasting national wind or solar power production each country or state is typically divided into smaller geographical areas. Within each area, there can be several wind and solar farms. Each farm can be further divided into individual wind turbines or solar panels. It is often beneficial for accuracy to reconcile forecasts for multiple levels of such a hierarchy rather than forecasting production for the whole country at once or simply aggregating forecasts for each individual turbine or panel into a production forecast for the country as a whole.

Figure 3 illustrates small examples of spatial and temporal hierarchies for quarter-hourly data. The spatial hierarchy with the country of Denmark at the top is divided into a Western and Eastern price area at the bottom denoted by DK1 and DK2, respectively. The temporal hierarchy has three levels, where the full hour at the top splits into a first and second half-hour, and each half-hour splits into two quarters at the bottom level. Furthermore, the spatial and temporal hierarchies can be combined into a spatiotemporal hierarchy, where inside each node of the spatial hierarchy there is a temporal hierarchy for that area.

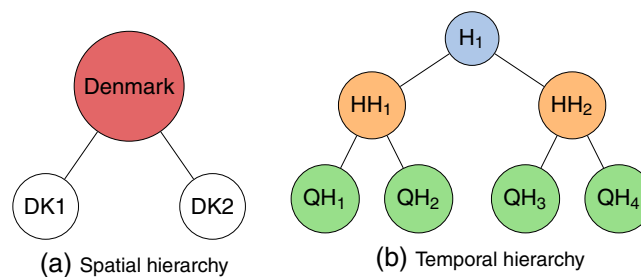


FIGURE 3 Spatial and temporal hierarchies for quarter-hourly, half-hourly, and hourly data for price areas DK1 and DK2 in Denmark



### 3.1 | Hierarchical forecasts

Hierarchies are defined by linear constraints. Given  $n$  individual *base* forecasts stacked in a column vector  $\hat{\mathbf{y}} \in \mathbb{R}^n$ , forecast reconciliation is concerned with finding *reconciled* forecasts  $\tilde{\mathbf{y}} \in \mathbb{R}^n$ , which are coherent. Reconciliation is needed when base forecasts  $\hat{\mathbf{y}}$  do not satisfy the aggregation constraints. As a side benefit, reconciled forecasts are often more accurate.

To represent the aggregation constraints, we introduce a summation matrix  $\mathbf{S}$  of order  $n \times m$ . For example, the temporal hierarchy in Figure 3b with total dimension  $n = 7$  has  $m = 4$  quarter-hourly forecasts at the bottom level and summation matrix

$$\mathbf{S} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (23)$$

The base forecasts can be constructed using the methods described in Section 2, or any other method. When reconciling forecasts across a hierarchy, it is typically not necessary to use very complex models for generating the individual base forecasts. However, it can be advantageous to use different methods to construct the base forecasts for the different levels of the hierarchy, similarly to combined forecasting (see Section 2.4). Naturally, this rarely leads to forecasts that are coherent.

The simplest way to produce coherent forecasts for a hierarchy is to either aggregate forecasts from the bottom level all the way up to the top level or disaggregate forecasts from the top level down to the bottom level. For many (energy) time series, it is possible to get considerably higher accuracy by combining forecasts from multiple aggregation levels to exploit information differences and mitigate model uncertainty through reconciliation (Nystrup et al., 2021). A straightforward way of achieving this is to construct forecasts for the entire hierarchy based on base forecasts from each level using bottom-up and top-down approaches and then combine these, as shown by Hollyman et al. (2021) and Di Fonzo and Girolimetto (2022).

The frameworks for combined forecasting and forecast reconciliation are similar in that they both incorporate information from different forecasts, either by combining point forecasts or by reconciling information from different aggregation levels in a hierarchy in an optimal way. The difference is that forecast reconciliation can handle forecasts on different temporal/spatial scales.

### 3.2 | Least-squares reconciliation

Recent studies on forecast reconciliation have focused on different least-squares estimators. As proven by Wickramasuriya et al. (2019), the reconciled forecasts which minimize the sum of the variances of the reconciled forecast errors are given by the generalized least-squares (GLS) estimator

$$\tilde{\mathbf{y}} = \mathbf{S}(\mathbf{S}^T \boldsymbol{\Sigma}^{-1} \mathbf{S})^{-1} \mathbf{S}^T \boldsymbol{\Sigma}^{-1} \hat{\mathbf{y}}, \quad (24)$$

where  $\boldsymbol{\Sigma}$  is the covariance of the base forecast errors.

Optimal forecast reconciliation is about finding a good estimator or approximation of the covariance matrix of the base forecast errors. Uncertainties propagate from the covariance matrix estimation to the reconciliation weights. Although the reconciled forecast will be coherent as long as the reconciliation weights meet the coherency constraints, they might not be optimal, as shown by Pritularga et al. (2021).

Hyndman et al. (2011) were among the first to apply the ordinary least-squares (OLS) estimator to compute reconciled forecasts for a spatial hierarchy. By approximating  $\boldsymbol{\Sigma}$  by a diagonal matrix, they circumvented the problem of

estimating the covariance of the base forecast errors. Hyndman et al. (2016) later proposed the use of weighted least squares (WLS), in order to take account of the variances on the diagonal of  $\Sigma$  while ignoring the off-diagonal covariances. Athanasopoulos et al. (2017) were first to apply forecast reconciliation to a temporal hierarchy. They considered three diagonal approximations of  $\Sigma$ .

### 3.3 | Reconciliation of wind and solar power forecasts

Zhang and Dong (2018) documented the benefit from taking into account correlations when reconciling short-term wind power forecasts across several wind farms. In many energy applications, the information embedded in the correlation structure in both space and time is useful for improving the accuracy of reconciled forecasts. The base forecasts, which can be generated independently using different methods, can be connected through a nondiagonal  $\Sigma$ .

In two successive articles, Yang et al. (2017a) and Yang et al. (2017b) applied a spatial and temporal hierarchy, respectively, for reconciling day-ahead solar power forecasts. In the structural case, they followed Wickramasuriya et al. (2019) by considering both diagonal and nondiagonal covariance estimators, with the latter leading to the largest accuracy improvements. In the temporal case, they followed Athanasopoulos et al. (2017) by disregarding potential information in the autocorrelation structure. Nystrup et al. (2020) extended the results of Wickramasuriya et al. (2019) to temporal hierarchies by showing the value of considering autocorrelations when reconciling day-ahead load forecasts.

It can be challenging to estimate the full covariance matrix for a hierarchy, which can easily be of very large dimension; yet it is difficult to know a priori which part of the error structure that is most important. To address these issues, Nystrup et al. (2021) proposed the use of eigendecomposition for dimensionality reduction. Meanwhile, Møller et al. (2021) applied statistical hypothesis testing to simplify the structure of  $\Sigma$  when reconciling forecasts.

Another issue that often arises in energy applications is that  $\Sigma$  in (24) changes with the season or is nonstationary. Thus, it can be necessary to apply adaptive estimation approaches, as suggested by Bergsteinsson et al. (2021) and Di Modica et al. (2021) in applications to heat load and wind power forecasting, respectively.

In spatiotemporal hierarchies, the dimension renders estimation of  $\Sigma$  infeasible for all practical purposes. So far studies have focused on separating the spatial and temporal dimensions and applying sequential approaches. In an application to solar power forecasting, Yagli et al. (2019) applied a temporal-then-spatial reconciliation approach. Unlike their approach, the sequential approaches proposed by Kourentzes and Athanasopoulos (2019) and Di Fonzo and Girolimetto (2021) guarantee that the reconciled forecasts are coherent in both space and time simultaneously. As for wind and solar power forecasting, the information embedded in the spatiotemporal correlation should be useful for further improving the accuracy of reconciled forecasts in future work by considering the spatial and temporal dimensions simultaneously rather than separately.

### 3.4 | Reconciliation of probabilistic forecasts

Several approaches have been proposed for extending reconciliation from the setting of point forecasting to probabilistic forecasting. Taieb et al. (2017, 2021) produced load forecasts for individual electricity consumers at the bottom to the total grid at the top of a spatial hierarchy. Their algorithm imposed dependencies between forecast distributions using samples from the probabilistic base forecasts from univariate models for the bottom level, which were reordered to match the empirical copula of residuals and aggregated bottom-up.

Jeon et al. (2019) constructed fully probabilistic wind power and load forecasts by reconciling a large number of forecasted quantiles in temporal hierarchies. Yang (2020) used a block-bootstrapping approach to reconcile probabilistic forecasts of solar power generation in a spatial hierarchy. Panagiotelis et al. (2022) showed how a projection derived from the estimator of Wickramasuriya et al. (2019) can be used to obtain a reconciled density analytically. They considered an application for forecasting Australian electricity generation from different energy sources.

## 4 | MULTIVARIATE PROBABILISTIC FORECASTING USING SDES

In this section, we consider probabilistic forecasting. Contrary to the point predictions reviewed in Section 2, probabilistic forecasts assign a probability to all possible outcomes. Meteorological ensemble forecasts can be used to generate

ensembles of future wind and solar power generation, and by a simple ranking probabilities can be assigned to the outcomes. Nielsen et al. (2006b) demonstrated that a statistical model is needed to ensure that the resulting probabilistic forecasts reliably span the observed forecast errors.

Univariate probabilistic forecasts take various forms, for example, quantiles or full (conditional) densities. A good introduction to classical probabilistic forecasting is found in Gneiting and Katzfuss (2014). Historically, methods related to a second-order moment or Gaussian assumption have been used to specify prediction intervals. However, depending on the forecast horizon, the conditional distribution may not be symmetric nor unimodal (Hyndman, 1995).

The conditional probability density function for wind and solar power production is negatively skewed for production near the installed capacity and positively skewed for low generation. Due to the fact that power production is double-bounded between zero and installed capacity, the beta-distribution has been suggested for short-term wind power forecasting (Luig et al., 2001; Yuan et al., 2019).

Characteristics of the forecast uncertainty also vary as a function of explanatory variables. This dependency can, for example, be used to describe that the uncertainty is larger for westerly wind compared to easterly wind. Nielsen et al. (2006a) described methods for quantile regression with explanatory variables, while Møller et al. (2008) suggested methods for adaptive quantile regression.

For many of the decision-making problems related to wind and solar power integration, such as start and stop of production facilities, block bidding in electricity markets, and optimal use of energy storage solutions, it is not sufficient to look at a single horizon. A simple approach is to use correlation or precision matrices or correlation functions to describe the persistence of forecast errors after suitable transformation (Nielsen & Madsen, 2002; Pinson et al., 2010). However, a correct specification of the persistence in the sequence of forecast errors requires multivariate probabilistic forecasts.

In this section, we focus on multivariate probabilistic forecasting using Stochastic Differential Equations (SDEs). In recent years, SDEs for wind and solar power forecasting have attracted considerable attention. The SDE framework is flexible as boundary conditions on both mean value structures and stochastic elements can be set up in an intuitive way. Using SDEs the complex structure of multivariate probabilistic forecasts can be described using a relatively small set of parameters.

As pointed out in Section 3, good estimates of the covariance matrix between and within forecasts are important for the accuracy of the reconciled forecasts. For forecast reconciliation, the covariance matrix in (24) contains covariances both within and between levels. In Section 4.1 below, we focus on the within-level covariance structure.

The within-level covariance, or more generally interdependence structure, is important by itself when forecasting, since error propagation is often governed by strong autocorrelation, in particular when sampling at high frequency. Such correlation structures are important, for example, for scenario generation. Moreover, in applications to wind and solar power forecasting, variances are highly dependent on the forecasted level due to the double-bounded nature of the problems.

## 4.1 | Stochastic differential equations

The usage of SDEs is no new development within forecasting. SDEs are used within a number of areas, including mathematical finance (see, e.g., Mikosch, 1999) and physics (see, e.g., Kampen, 2007). Recently, some specialized methods for wind and solar power forecasting have been introduced, including novel ways of using SDEs to reduce and quantify uncertainty of forecasts.

The background and theory behind the use of SDEs for forecasting is well known. Hence, we only provide a brief introduction to SDEs and their use in forecasting and refer to Øksendal (1985) for a more thorough walk-through of the theoretical aspects.

When applied in forecasting, SDEs are generally expressed as state-space models of the form

$$d\mathbf{X}_t = f(\mathbf{X}_t, \mathbf{u}_t, t, \boldsymbol{\theta})dt + \sigma(\mathbf{X}_t, \mathbf{u}_t, t, \boldsymbol{\theta})d\mathbf{W}_t, \quad (25)$$

$$\mathbf{y}_{t_k} = h(\mathbf{X}_{t_k}, \mathbf{u}_{t_k}, t_k, \boldsymbol{\theta}) + \mathbf{e}_{t_k}. \quad (26)$$

This is known as a continuous-discrete time model, which implies that the observations  $\mathbf{y}_{t_k}$  are observed at discrete time points  $t_k$ , while the states  $\mathbf{X}_t$  assume values in continuous time.

The SDE (25) itself is essentially an ordinary differential equation (ODE) with a stochastic element added to it, with  $W_t$  being a Wiener process. The function  $f(\cdot)$ , known as the drift term, describes the mechanics of the system, similarly to the way dynamical systems often are described by ODEs. The stochastic element is described by the function  $\sigma(\cdot)$ , known as the diffusion term, which models the noise pattern of the system. Both of these functions can include input variables  $u_t$  describing, for example, an existing point forecast or NWP. In a state-space model, the SDE is accompanied by an observation equation (26), which, in addition to separating the observation noise from the system noise, models the link between the system state and the observations.

As mentioned above, wind and solar power are naturally bounded between zero and the installed capacity. Such restrictions should be included directly in the system equation. For example, if the state  $X_t$  represents the weather (usually wind) and  $y_{t_k}$  is the power output, then the double-bounded nature is naturally built into the observation equation  $h(\cdot)$  (see Iversen et al., 2017a). If the state represents power output, then the restriction is naturally imposed through the diffusion term (see Møller et al., 2016, and the example below).

With this sort of versatility, SDEs are a suitable framework to describe the physics of wind and solar power production. Since the physical aspects of the systems are often described by ODEs, these can be adapted to the drift term of an SDE. For wind power forecasting this could, for example, be a power curve model based on meteorological forecasts. This leaves the diffusion term where the stochastic behavior of the system can be modeled. Since the stochastic behavior can be modeled independently, this also means that SDEs can be used to convert point forecasts into probabilistic forecasts by using the point forecast as a drift term and then modeling the uncertainty of the point forecast in the diffusion term.

#### 4.1.1 | Example: A logistic SDE for wind power

Bjerregård et al. (2021) provide an example of an SDE for simulating wind power production in their second case study. They simulate normalized wind power production by a bounded point-forecast-driven logistic SDE

$$dY_t = \theta(\hat{y}_t - Y_t)dt + \sigma Y_t(1 - Y_t)dW_t. \quad (27)$$

This has a mean-reverting property in the drift term, which forces the state value  $Y_t$  to approach the point forecast  $\hat{y}_t$  at a rate determined by the parameter  $\theta$ . The mean-reverting drift term is used in many studies (see, e.g., Møller et al. (2016) or Badosa et al. (2018)).

The diffusion term ensures that the observations produced by the simulation are bounded between zero and one. The boundedness of the forecast is essential for producing realistic and reliable power forecasts; however, the way in which this is achieved differs between studies. Most studies choose to bound their forecast when modeling the diffusion term. For instance, Møller et al. (2016) bounded the noise based on both the state value and the forecast using a logistic-type SDE. Other methods have been proposed, including by Iversen et al. (2017b) who bounded their forecasts in the observation equation (26), thus freeing up the diffusion term for modeling.

To examine the predictive performance of the SDE in (27), we apply it to some real-life forecasts from a Danish offshore wind farm in Klim. We use the following parameters:

$$\theta = 2 \quad \sigma = 0.8$$

As illustrated in Figure 4, the predictive distribution narrows and widens following the point forecasts. This is a direct result of the parametrization of the SDE. The mean-reverting property of the drift term ensures that the state follows the point forecast, while the parametrization of the diffusion term ensures that the predictive distribution narrows greatly when the forecast approaches the boundary. This is just a simple example and more advanced modeling approaches definitely exist; but it emphasizes how powerful a modeling tool SDEs are when creating probabilistic forecasts.

## 4.2 | Wind and solar power forecasting using SDEs

SDEs have been used for a number of different purposes in recent studies on wind and solar power forecasting. A number of studies have gone the route of producing probabilistic forecasts of power generation by modeling and quantifying the uncertainty of point forecasts. Møller et al. (2016) established the general framework for using SDEs for wind power

forecasting and proposed a logistic-type SDE with state-dependent diffusion to model wind power production from a Danish off-shore wind farm. They showed the importance of modeling the conditional correlation structure, in addition to deriving Lamperti-transformed models and second-order moment representations.

Elkantassi et al. (2017) extended the work of Møller et al. (2016) by considering a parametric form for the drift coefficient. Iversen et al. (2017b) took a different approach by not including a premade point forecast, instead constructing a dynamic power curve model to use in the observation equation while modeling wind speed as the underlying stochastic process. Caballero et al. (2021) built upon the work of Møller et al. (2016) and Elkantassi et al. (2017) by introducing derivative tracking into the drift term.

Similar developments have been made within solar power forecasting. Iversen et al. (2014) proposed an SDE approach for modeling solar irradiance. They introduced a fairly simple model and proposed mechanistic extensions as well as extensions based on a time series analysis perspective.

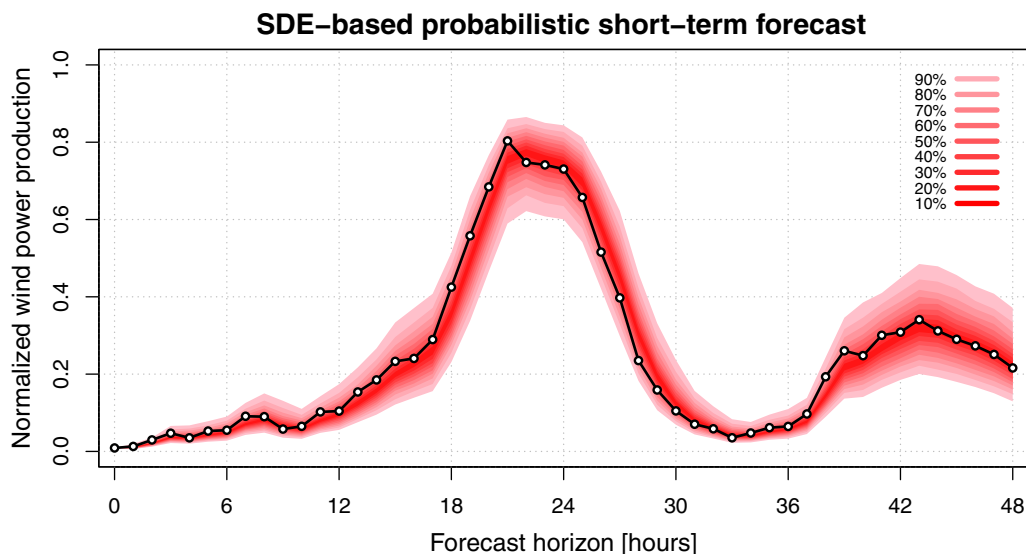
Iversen et al. (2017a) provided some suggestions for how spatiotemporal solar power forecasting could be done by what is effectively stochastic partial differential equations (SPDEs) using the SDE modeling framework described by Kristensen et al. (2004). They achieved this by imposing a grid structure on a solar power plant and solving an SDE in every grid point based on the neighboring PV panels.

Badosa et al. (2018) applied similar modeling approaches as Møller et al. (2016) used for wind power forecasting to a solar power forecasting application. SDEs have also been used in hybrid methods with other forecasting tools, such as Zhang and Kong (2021) who used SDEs alongside recurrent neural networks to forecast solar power production.

Pinson et al. (2009) introduced the concept of transforming observed wind power processes to stationary standard normally distributed processes using probabilistic forecasts and copulas. They implemented this with forecasts covering multiple horizons, hence a temporal model. It was later extended by Tastu et al. (2013) and Zhang et al. (2013) to include the spatial dimension as well. In the mentioned studies, the transformed processes were described by a covariance matrix and not more advanced models. Describing correlations using SDEs instead of a covariance matrix could open the door to a wide range of possibilities. So far, no one has explored this for wind and solar power forecasting. This would enable the introduction of inputs to the model, such as wind speed and direction for describing changing correlation patterns, without having to describe complicated parametric distributions directly in the SDEs, since this is taken care of at the copula transformation stage.

## 5 | FORECAST EVALUATION

After a forecast has been issued, it is natural to ask how good the forecast is or how well it performs compared to competing forecasts. The treatment of these questions is called forecast evaluation, which should be of concern to any



**FIGURE 4** Probabilistic forecast of normalized wind power up to 48 h ahead given the information available at  $t=0$ . The multivariate forecast with temporal dimension 48 is generated from the SDE in (27). Prediction intervals from 10% to 90% are shown

serious forecast practitioner. As touched upon in the introduction, financial gains in the renewable energy sector are extremely dependent on good forecasts. This is one reason why forecast evaluation is so important.

## 5.1 | Scoring rules

Consider any given forecasting scenario. Let  $Y$  be the variable subject to forecasting, let  $G$  be the forecast of  $Y$ , and let  $y$  be the realized value of  $Y$ , that is, the observation. Then, we define the scoring rule,  $S(G, y)$ , to be a function that measures the loss associated with the use of  $G$  when the actual realization is  $y$  (Gneiting & Raftery, 2007). The lower the value of the scoring rule is, the better the forecast is considered to be. The overall score  $\bar{S}$  of a forecasting model can then, for example, be the average of the scores of all individual pairs of forecasts and observations. With  $N$  pairs that would be

$$\bar{S}(G, y) = \sum_{i=1}^N S(G_i, y_i). \quad (28)$$

The most well-known example of a scoring rule is the root mean square error,

$$\text{RMSE}(G, y) = \sqrt{\sum_{i=1}^N \frac{1}{N} (y_i - \hat{y}_i)^2}. \quad (29)$$

The RMSE measures the standard deviation of the forecast error. Minimizing RMSE ensures that the forecast is *calibrated*, that is, that the average forecast is unbiased with respect to the observations. Alternatively, the calibration of a forecast can be verified qualitatively by inspecting its rank histogram (Hamill, 2001), which is also applicable to multivariate forecasts (Gneiting et al., 2008).

Calibration is one of the key features we care about when evaluating a forecast. If the forecast is probabilistic, then the *sharpness* of the forecast density is also important. Sharper, that is, narrower, forecast densities are preferred. The Continuous Ranked Probability Score (CRPS) is an example of a scoring rule for probabilistic forecast evaluation that takes both sharpness and calibration into account (Matheson & Winkler, 1976). It is defined as

$$\text{CRPS}(F, y) = \int_{-\infty}^{\infty} (F(u) - \mathbb{I}(u \geq y))^2 du, \quad (30)$$

where  $F$  is the cumulative distribution function (CDF) of the forecast. The CRPS is commonly applied in the context of probabilistic forecasting of wind and solar power (see, e.g., Andrade & Bessa, 2017; Iversen et al., 2017a; Guan et al., 2020). Sometimes a generalized version of the CRPS, known as the energy score, is used (Gneiting & Raftery, 2007).

If the forecast is multivariate, then a correctly specified *correlation* structure is crucial. In theory, it is possible to evaluate all these three key features—calibration, sharpness, and correlation—of a multivariate, probabilistic forecast in one suitable scoring rule. The logarithmic score (LogS), is defined as the negative log-likelihood of the multivariate probability density function (PDF)  $\mathbf{f}$ , given the multivariate observation  $\mathbf{y}$ :

$$\text{LogS}(\mathbf{f}, \mathbf{y}) = -\log \mathbf{f}(\mathbf{y}). \quad (31)$$

The practical issue with the logarithmic score is the “curse of dimensionality.” Unless an explicit expression for the forecast density is available, the runtime and memory footprint of the estimation of the  $N$ -dimensional density quickly explodes. Instead, for evaluation of the correlation structure of a multivariate forecast, we recommend the Variogram Score of order  $p$  (VarS; Scheuerer & Hamill, 2015):

$$\text{VarS}_p(\mathbf{f}, \mathbf{y}) = \sum_{i=1}^{k-1} \sum_{j=i+1}^k w_{ij} \left( |\mathbf{y}_i - \mathbf{y}_j|^p - [E|\mathbf{X}_i - \mathbf{X}_j|^p] \right)^2, \quad (32)$$

where for each  $i = 1, \dots, k$ ,  $\mathbf{X}_i$  is a vector of sampled values from the marginal forecast density  $f_i$ , and the weights  $w_{ij}$  are chosen by the user in a way that reflects how related individual pairs are expected to be. If there is no such presumption, then identity weights can be used. It is common to set  $p = \frac{1}{2}$  in order for the VarS to have good sampling properties (Bjerregård et al., 2021). The VarS has been shown to be very effective at separating models with correctly specified correlation structure from those with a wrongly specified one. Furthermore, the VarS is very scalable with respect to runtime and memory footprint, even for dimensions in the hundreds. The major downside of VarS is that it cannot separate models with respect to calibration. To the best of our knowledge, no extended version of the VarS has been published to accommodate this deficiency. Hence, the best option currently available is to evaluate the correlation structure of the multivariate density with VarS, and then evaluate the calibration and sharpness of the marginal densities with either CRPS or the univariate version of LogS (Bjerregård et al., 2021). In the case of a contradiction, where one model is superior with respect to the VarS and a competing model is superior with respect to the LogS or CRPS, the user must decide which feature to prioritize for their particular evaluation problem. As a relatively new scoring rule, the VarS has not seen as much application as the CRPS yet, although examples of application to wind power forecasts do exist (see, e.g., Staid et al., 2017). Being tailored towards multivariate forecasts, its relevance is expected to increase in the future due to the growing demand for multivariate forecasting solutions (Gneiting & Katzfuss, 2014).

### 5.1.1 | Example: Evaluation of a probabilistic wind power forecast

In the following example, we demonstrate a practical application of forecast evaluation. Consider the SDE-based model in (27) used for forecasting of the wind power series in Figure 4. This model is driven by a series of point forecasts  $\hat{y}_{t+k|t}$  available in the Klim data set. We can construct a competing forecasting model by sampling boundary-reflecting Gaussian distributions around the point forecasts with  $\mu = \hat{y}_{t+k|t}$  and  $\sigma = 0.1$ . For evaluation, CRPS and VarS with  $w_{ij} = 1/|i-j|$  are applied.

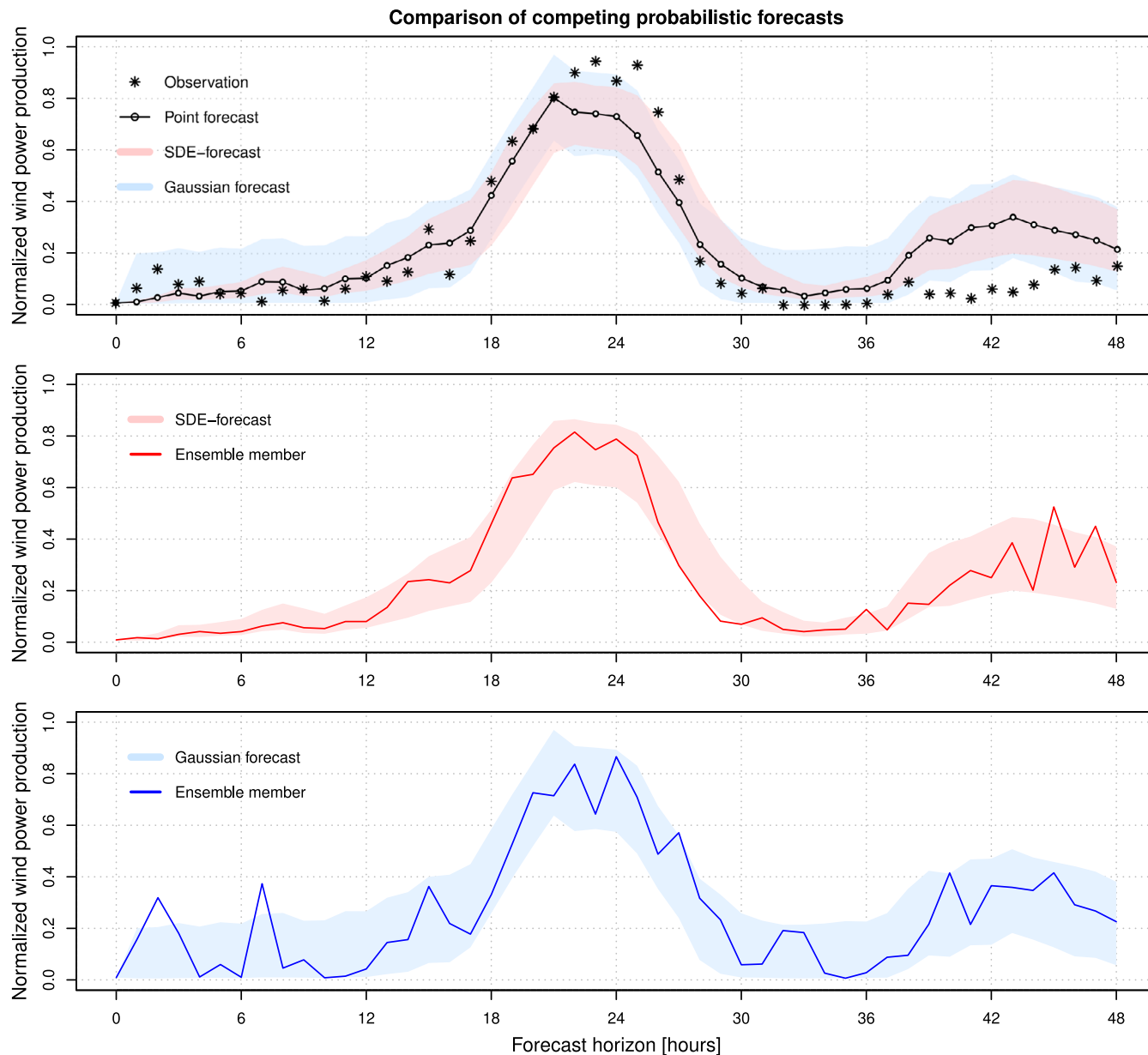
A visual comparison of the two forecasting models is shown in Figure 5, and the resulting scores are listed in Table 1. It is seen that the Gaussian model is superior according to the CRPS, which can be explained by its wider confidence intervals covering the observations better than the seemingly too narrow SDE forecast. On the other hand, the SDE model is superior according to the VarS, which can be explained by its inherent autocorrelation that mimics the autocorrelation of the observations better than randomly sampling around the point forecasts, as is seen in the center and bottom graphs in Figure 5. These conclusions should not be attributed to those two model classes in general. The important message is that different metrics reward and penalize different features of forecasts.

## 5.2 | Evaluation of wind and solar power forecasts in practice

There are several aspects of evaluation of wind and solar power forecasts in practice that are worth discussing in relation to large-scale integration of renewable energy sources. While characteristics such as maximum forecast horizon and resolution in space and time are relatively easily identified, identification of the required statistical characteristics demands more thorough analyses. Such analyses are based on the formulation of the problem in a random variable setting, where the expected revenue or imbalance cost can be optimized conditional on the information available at forecast time.

Generally, in situations where each time step is handled separately, as is normal in electricity markets, quantile forecasts or single-horizon probabilistic forecasts of power production are sufficient. In case of asymmetric cost functions, reliable estimates of the single-horizon conditional density are needed and, hence, the use of CRPS might be optimal. However, such forecasts do not contain information about the temporal development of the future power production. Consequently, if issues related to storage capacity, start-up time, or costs are taken into account, then more advanced forecasts and evaluation are required. In those cases, the evaluation of forecasts using the Variogram Score is important.

If the forecasts are based on SDEs, then the optimal decision could be taken using stochastic programming based on scenarios of the future power production. The scenarios can, together with the currently planned power production, be used to calculate probabilities of imbalance events, which are more relevant from a system operation point of view. Scenarios for the future power production from renewable energy sources, or deviations of these from the current plan, can also be used to address the required buffering capacity, for example, obtained by letting hydropower production deviate from the current plan. A rather comprehensive discussion about how to select the most suitable forecast solution for different large-scale wind integration case studies can be found in Nielsen et al. (2011).



**FIGURE 5** Top: Comparison of the two probabilistic forecasts in the example in Section 5.1.1 shown together with the original point forecasts as well as the observations. Center: The probabilistic SDE-forecast with one of its ensemble members displayed. Bottom: The probabilistic Gaussian forecast with one of its ensemble members displayed

**TABLE 1** Resulting scores from the forecast evaluation of the two competing probabilistic forecasts in the example in Section 5.1.1

	CRPS	VarS
SDE	0.0731	3.0025
Gaussian	0.0706	3.4209

## 6 | CONCLUSION AND OUTLOOK

We have highlighted methods for adaptive, conditional parametric, and combined forecasting. We consider these methods to be some of the cornerstones of modern wind and solar power forecasting. Recent developments in



multivariate wind and solar power forecasting have been rapid. We have reviewed some of these developments within reconciliation of hierarchical forecasts, forecasting using SDEs, and forecast evaluation.

The methods reviewed are characterized by a low computational burden, allowing for implementations in near real-time solutions. Consequently, many of the methods described are core elements in some of today's most widely used tools for operational wind and solar power forecasting. This is different from many of the computationally intensive machine learning methods that often perform well in forecasting competitions, which are less suited for real-time production environments. Additionally, the methods reviewed are less sensitive, i.e., to initialization, and can easily be made adaptive, which is crucial to operational success.

Reconciliation of hierarchical forecasts is a relatively new topic that has garnered a lot of interest from forecasters due to the improvements found; however, there are still paths to be explored. As discussed above, spatiotemporal hierarchies show a lot of promise although, among others, issues with dimensionality have to be solved for them to become useful in practice. The methods applied for reconciling probabilistic forecasts differ from study to study, and it seems that no consensus has been found in terms of how this should be done. Ongoing research is exploring ways to embed hierarchical structure into machine learning models to get coherent forecasts. Machine learning methods often require variables that are uniformly scaled, which is a challenge in hierarchical forecasting, where data from different aggregation levels naturally has different scales.

Studies using SDEs for forecasting wind and solar power production have developed increasingly specialized models for the purpose, and have shown how useful the method can be. SDEs are powerful, yet often overlooked, models that can be applied in many cases where probabilistic forecasts are wanted. SDEs have many desirable properties in terms of model building, propagation of noise and other effects, and imposing restrictions on the state space.

The non-linear SDEs needed for wind and solar power forecasting are computationally challenging to estimate, which has made adaptive methods difficult to apply. With the development of packages using automatic differentiation, considerable improvement in terms of speed can be obtained (see, e.g., Kristensen et al., 2016). Future work should consider adaptive estimation in nonlinear SDEs for forecasting of wind and solar power, similarly to recent developments in other fields of application (see Stentoft et al., 2021, for an example from wastewater treatment).

As we have shown, the choice of method for forecast evaluation is important, and especially so for multivariate probabilistic forecasts. Depending on the method of choice, the forecast can be misrepresented or features can be overlooked. In practice, it often requires thorough analysis of the actual decision problem to determine the most appropriate evaluation criteria based on the required statistical characteristics. The interplay between forecasting model, decision problem, and evaluation criteria continues to be an important problem, both from an academic and practical perspective, well worth further investigation.

## AUTHOR CONTRIBUTIONS

**Mikkel L. Sørensen:** Conceptualization (equal); data curation (equal); formal analysis (equal); investigation (equal); methodology (equal); project administration (equal); resources (equal); software (equal); supervision (equal); validation (equal); visualization (equal); writing – original draft (equal); writing – review and editing (equal). **Peter Nystrup:** Conceptualization (equal); data curation (equal); formal analysis (equal); investigation (equal); methodology (equal); validation (equal); visualization (equal); writing – original draft (equal); writing – review and editing (equal). **Mathias Bjerregård:** Conceptualization (equal); data curation (equal); formal analysis (equal); investigation (equal); methodology (equal); resources (equal); software (equal); validation (equal); visualization (equal); writing – original draft (equal); writing – review and editing (equal). **Jan Møller:** Conceptualization (equal); data curation (equal); formal analysis (equal); investigation (equal); methodology (equal); software (equal); supervision (equal); validation (equal); visualization (equal); writing – original draft (equal); writing – review and editing (equal). **Peder Bacher:** Conceptualization (equal); investigation (equal); supervision (equal); validation (equal); writing – original draft (equal); writing – review and editing (equal). **Henrik Madsen:** Conceptualization (equal); formal analysis (equal); funding acquisition (equal); methodology (equal); project administration (equal); resources (equal); supervision (equal); validation (equal); writing – original draft (equal); writing – review and editing (equal).

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## CONFLICT OF INTEREST

The authors have declared no conflicts of interest for this article.

## DATA AVAILABILITY STATEMENT

Data available on request from the authors.

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